F.E. (old) AM - I Date 13/5/14

Applied Maths - I

(OLD COURSE) **QP Code : MV-17654**

(3 Hours) **[Total Marks :100**

N.B. (1) Question no. **1** is **compulsory.**
(2) Attempt any **four** questions from Q.no. **2** to Q.no.7.
(3) Answer to **subquestion** should be written **together.**

1. (a) If $(1+\cos\theta+i\sin\theta)(1+\cos2\theta+i\sin2\theta)=u+iv$ proove that $u^2+v^2=16\cos^2\frac{\theta}{2}\cos^2\theta$ **3**

(b) If $y=\sin ax+\cos ax$ then P.T.
$y_n=a^n[1+(-1)^n\sin 2ax]^{1/2}$ **3**

(c) If $\frac{d\bar{a}}{dt}=\bar{u}\times\bar{a}$, $\bar{u}\times\bar{b}=\frac{d\bar{b}}{dt}$ then S.T. $\frac{d}{dt}(\bar{a}\times\bar{b})=\bar{u}\times(\bar{a}\times\bar{b})$ **3**

(d) P.T. $\cos^2 x=1-x^2+\frac{1}{3}x^4-\frac{2}{45}x^6+.....$ **3**

(e) If $u=\frac{1}{r}$ where $r=\sqrt{x^2+y^2+z^2}$ then find $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}$ **4**

(f) Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is minimum. **4**

2. (a) If $|z_1+z_2|=|z_1-z_2|$. Show that $\frac{z_2}{z_1}$ is purely imaginary. **6**

(b) Separate into real and imaginary parts of $\cos^{-1}\left(\frac{3i}{4}\right)$ **6**

(c) If $u=x^n f_1\left(\frac{y}{x}\right)+y^n f_2\left(\frac{x}{y}\right)$ then find $x^2\frac{\partial^2u}{\partial x^2}+2xy\frac{\partial^2u}{\partial x\partial y}+y^2\frac{\partial^2u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ **8**

3. (a) Test whether C.M.V.T. holds for $f(x)=x^2+2$ and $g(x)=x^3-1$ in (1, 2) and if so fin C. **6**

(b) Find scalars p and q if **6**

$(\bar{a}\times\bar{b})\times\bar{c}=\bar{a}\times(\bar{b}\times\bar{c})$ where

$\bar{a}=2\hat{i}+\hat{j}+p\hat{k}$

$\bar{b}=\hat{i}-\hat{j}$

$\bar{c}=4\hat{i}+q\hat{j}+2\hat{k}$

(c) Expand by $(1+x+x^2+x^3)$ in powers of a upto term x^8. **8**

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4. (a) Find roots of $x^{14} + 127\,x^7 - 128 = 0$. 6

(b) Test the convergence of $\sum \frac{3^n + 4^n}{4^n + 5^n}$ 6

(c) If, $y^{1/m} + y^{-1/m} = 2x$ S.T. $(x^2-1)y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2 - m^2)\,y_n = 0$. 8

5. (a) Find n^{th} derivative of $y = \frac{x^4}{(x-1)(x-2)}$ 6

(b) Find a and b if $\lim_{x \to 0} \frac{x(1 + a\cos x) - b\sin x}{x^3} = 1$ 6

(c) Find divergence $\bar{F}$ and curl $\bar{F}$ 8

where $\bar{F} = \frac{x\,\hat{i} - y\,\hat{j}}{x^2 + y^2}$

6. (a) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ 6

(b) Find directimal derivativ of $\phi = 4xz^3 - 3x^2y^2z$ at (2, -2, 2) in the direction $\nabla(x^2y^2z^2)$ at (1, -1, 1). 6

(c) Show that 8

$$\tan^{-1}\left(e^{i\theta}\right) = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right) - \frac{i}{2}\log\left[\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right]$$

7. (a) If $u = t^n\, e^{-r^2/4t}$ then find the value of n so that $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t}$ 6

(b) Using theory of approximation find approximate value of $f(3, 4) - f(2.9, 4.1)$ where $f(x, y) = (50 - x^2 - y^2)^{1/2}$ 6

(c) If $i^{i^{i \cdot \cdot \cdot \infty}} = Z$ where $z = x + iy$ prove that $|z|^2 = e^{-(4n+1)\pi y}$ 8
