

Applied Maths - I**(OLD COURSE)****QP Code : MV-17654**

(3 Hours)

[Total Marks : 100

- N.B.** (1) Question no. 1 is **compulsory**.
 (2) Attempt any **four** questions from Q.no. 2 to Q.no.7.
 (3) Answer to **subquestion** should be written **together**.

1. (a) If $(1 + \cos\theta + i\sin\theta)(1 + \cos 2\theta + i\sin 2\theta) = u + iv$ prove that $u^2 + v^2 = 16\cos^2 \frac{\theta}{2} \cos^2 \theta$ 3
 (b) If $y = \sin ax + \cos ax$ then P.T. 3

$$y_n = a^n [1 + (-1)^n \sin 2ax]^{1/2}$$

 (c) If $\frac{d\bar{a}}{dt} = \bar{u} \times \bar{a}$, $\bar{u} \times \bar{b} = \frac{d\bar{b}}{dt}$ then S.T. $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{u} \times (\bar{a} \times \bar{b})$ 3
 (d) P.T. $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \dots$ 3
 (e) If $u = \frac{1}{r}$ where $r = \sqrt{x^2 + y^2 + z^2}$ then find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ 4
 (f) Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is minimum. 4
2. (a) If $|z_1 + z_2| = |z_1 - z_2|$. Show that $\frac{z_2}{z_1}$ is purely imaginary. 6
 (b) Separate into real and imaginary parts of $\cos^{-1}\left(\frac{3i}{4}\right)$ 6
 (c) If $u = x^{n_1} f_1\left(\frac{y}{x}\right) + y^{n_2} f_2\left(\frac{x}{y}\right)$ then find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ 8
3. (a) Test whether C.M.V.T. holds for $f(x) = x^2 + 2$ and $g(x) = x^3 - 1$ in $(1, 2)$ and if so find C. 6
 (b) Find scalars p and q if 6

$$(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$$
 where

$$\bar{a} = 2\hat{i} + \hat{j} + p\hat{k}$$

$$\bar{b} = \hat{i} - \hat{j}$$

$$\bar{c} = 4\hat{i} + q\hat{j} + 2\hat{k}$$

 (c) Expand by $(1 + x + x^2 + x^3)$ in powers of x upto term x^8 . 8

8

[TURN OVER**Con. 9412-14.**

4. (a) Find roots of $x^{14} + 127x^7 - 128 = 0$. 6
- (b) Test the convergence of $\sum \frac{3^n + 4^n}{4^n + 5^n}$ 6
- (c) If, $y^{1/m} + y^{-1/m} = 2x$ S.T. $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$. 8
5. (a) Find n^{th} derivative of $y = \frac{x^4}{(x-1)(x-2)}$ 6
- (b) Find a and b if $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$ 6
- (c) Find divergence \bar{F} and curl \bar{F} 8
 where $\bar{F} = \frac{x\hat{i} - y\hat{j}}{x^2 + y^2}$
6. (a) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ 6
- (b) Find directiml derivativ of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -2, 2)$ in the direction $\nabla(x^2y^2z^2)$ at $(1, -1, 1)$. 6
- (c) Show that 8
- $$\tan^{-1}(e^{i\theta}) = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right) - \frac{i}{2} \log \left[\tan \left(\frac{\pi}{4} - \frac{\theta}{2} \right) \right]$$
7. (a) If $u = t^n e^{-t^2/4t}$ then find the value of n so that $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t}$ 6
- (b) Using theory of approximation find approximate value of $f(3, 4) - f(2.9, 4.1)$ 6
 where $f(x, y) = (50 - x^2 - y^2)^{1/4}$
- (c) If $i^{i \cdot \infty} = z$ where $z = x + iy$ prove that $|z|^2 = e^{-(4n+1)\pi y}$ 8

