

Applied Maths - I**(OLD COURSE)****QP Code : MV-17654**

(3 Hours)

**[ Total Marks : 100**

- N.B. (1) Question no. 1 is compulsory.  
 (2) Attempt any **four** questions from Q.no. 2 to Q.no.7.  
 (3) Answer to **subquestion** should be written **together**.

1. (a) If  $(1 + \cos\theta + i\sin\theta)(1 + \cos 2\theta + i\sin 2\theta) = u + iv$  prove that  $u^2 + v^2 = 16 \cos^2 \frac{\theta}{2} \cos^2 \theta$  3  
 (b) If  $y = \sin ax + \cos ax$  then P.T.  
 $y_n = a^n [1 + (-1)^n \sin 2ax]^{1/2}$  3  
 (c) If  $\frac{d\bar{a}}{dt} = \bar{u} \times \bar{a}$ ,  $\bar{u} \times \bar{b} = \frac{d\bar{b}}{dt}$  then S.T.  $\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{u} \times (\bar{a} \times \bar{b})$  3  
 (d) P.T.  $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \dots$  3  
 (e) If  $u = \frac{1}{r}$  where  $r = \sqrt{x^2 + y^2 + z^2}$  then find  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$  4  
 (f) Divide 24 into three parts such that the continued product of the first, square of the second and cube of the third is minimum. 4
2. (a) If  $|z_1 + z_2| = |z_1 - z_2|$ . Show that  $\frac{z_2}{z_1}$  is purely imaginary. 6  
 (b) Separate into real and imaginary parts of  $\cos^{-1}\left(\frac{3i}{4}\right)$  6  
 (c) If  $u = x^{n_1} f_1\left(\frac{y}{x}\right) + y^{n_2} f_2\left(\frac{x}{y}\right)$  then find  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  8
3. (a) Test whether C.M.V.T. holds for  $f(x) = x^2 + 2$  and  $g(x) = x^3 - 1$  in  $(1, 2)$  and if so find C. 6  
 (b) Find scalars  $p$  and  $q$  if 6  
 $(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$  where  
 $\bar{a} = 2\hat{i} + \hat{j} + p\hat{k}$   
 $\bar{b} = \hat{i} - \hat{j}$   
 $\bar{c} = 4\hat{i} + q\hat{j} + 2\hat{k}$   
 (c) Expand by  $(1 + x + x^2 + x^3)$  in powers of  $x$  upto term  $x^8$ . 8

8

**[ TURN OVER****Con. 9412-14.**

4. (a) Find roots of  $x^{14} + 127x^7 - 128 = 0$ . 6
- (b) Test the convergence of  $\sum \frac{3^n + 4^n}{4^n + 5^n}$  6
- (c) If,  $y^{1/m} + y^{-1/m} = 2x$  S.T.  $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$ . 8
5. (a) Find  $n^{\text{th}}$  derivative of  $y = \frac{x^4}{(x-1)(x-2)}$  6
- (b) Find a and b if  $\lim_{x \rightarrow 0} \frac{x(1 + a \cos x) - b \sin x}{x^3} = 1$  6
- (c) Find divergence  $\bar{F}$  and curl  $\bar{F}$  8  
 where  $\bar{F} = \frac{x\hat{i} - y\hat{j}}{x^2 + y^2}$
6. (a) If  $u = f(e^{x-y}, e^{y-z}, e^{z-x})$  find  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$  6
- (b) Find directiml derivativ of  $\phi = 4xz^3 - 3x^2y^2z$  at  $(2, -2, 2)$  in the direction  $\nabla(x^2y^2z^2)$  at  $(1, -1, 1)$ . 6
- (c) Show that 8  

$$\tan^{-1}(e^{i\theta}) = \left(\frac{n\pi}{2} + \frac{\pi}{4}\right) - \frac{i}{2} \log \left[ \tan \left( \frac{\pi - \theta}{4} \right) \right]$$
7. (a) If  $u = t^n e^{-t^2/4t}$  then find the value of n so that  $\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) = \frac{\partial u}{\partial t}$  6
- (b) Using theory of approximation find approximate value of  $f(3, 4) - f(2.9, 4.1)$  6  
 where  $f(x, y) = (50 - x^2 - y^2)^{1/4}$
- (c) If  $i^{i \cdot \infty} = z$  where  $z = x + iy$  prove that  $|z|^2 = e^{-(4n+1)\pi y}$  8

